RATIONALITY TEST OF RATE-MAKING DECISION IN INSURANCE MARKET

Tsai-Jyh Chen*

陳 彩 稚*

摘 要

本文主要目的是對於保險人訂價決策是否依據理性預期原則作一實證研究。由於過去三十多年來美國產險業之核保利潤一直呈現周期性波動，因此學者對於保險人是否任意操縱保險價格每有爭論。然而過去的研究多僅就保險人訂價決策之理念作觀念性的討論，尚未能進行實證分析。本文則利用產險市場之實際資料，直接進行實證研究，以瞭解保險人是否以理性預期爲基礎制定保費。根據實證分析結果，本文認爲未能推翻保費是以理性預期爲基礎所制定之假設。

Abstract

The objective of this paper is to investigate empirically the process of insurance rate-making decision to see whether insurers set price based on rational expectations. Because of periodic fluctuations of underwriting profits in property-liability insurance industry, it has been a controversy whether insurance price is rationally set. Some of the previous literature argues that fluctuations result from irrational pricing decision. On the other hand, other studies suggest that profit fluctuations might be generated even if insurers set prices under rational expectations. However, these studies have not provided an empirical analysis of market rationality. Thus it is of interest to conduct a rationality test for pricing decision in insurance market. The findings of this paper suggest that market rationality hypothesis cannot be rejected for property-liability insurance industry.

I. INTRODUCTION

The periodic fluctuations of underwriting profits, the so-called underwriting cycles, in property-liability insurance have important impacts on insurance pricing, industry stability, and solvency. Since at least the 1950's, underwriting returns of property-liability insurers in the U.S. have been characterized by significant cyclical fluctuations.

*作者為本校保險系副教授

According to the pricing formulas in most of the traditional insurance literature,[1] underwriting return should be random deviation from expected return. The six-year cycles of underwriting returns presenting significant autocorrelation seem contradictory to ratemaking principles. Thus, most of the earlier studies were inclined to attribute the cycles to irrationality of insurers in setting prices in order to pursue investment income and operating ratios, e.g., Wilson (1981) and Stewart (1984). The arguments in these articles imply that insurance prices are set subjectively to fit financial purposes of the insurers rather than to compensate the expected losses of the insured.

On the other hand, recent research attributes cycles to market mechanism which are not under the control of individual insurer, e.g., Doherty and Kang (1988). Cummins and Outreville (1987) even directly hypothesize that insurers might be rational expectation and the cycles result from some institutional intervention factors.

The objective of this paper is to investigate empirically the process of insurance rate-making to see whether insurers set price based on rational expectations. Although some of the earlier literature argues that cycles result from irrational pricing decision and others suggest that cycles might be generated even pricing under rational expectations, these studies did not provide an empirical analysis of market rationality. Thus, it is of interest to conduct a rationality test for rate-making decision in insurance market.

The empirical analysis is based on private passenger automobile insurance because automobile insurance has the largest premium volume of all property-liability insurance lines. The remainder of this paper consists of the following sections. Section II presents testing hypothesis and model for rationality test, and methodology for conducting the test is in section III. Data description is provided in section IV. The empirical results are discussed in section V, then followed by conclusion and suggestions in section VI.

II. Testing Hypothesis and Model

The concept of rational expectations was initially proposed by Muth in 1961, and it has been extensively applied not only in macroeconomics (e.g. Barro, 1976) but also in other fields like finance (e.g. Attfield and Duck, 1982) and accounting (Givoly, 1985). On the other hand, both the testability and the test methodology of rational

[1] In most of earlier insurance literatures price is equal to expected loss E(L) and underwriting return is equal to the estimation error, L - E(L), which is usually assumed as independent random error.

expectations hypothesis have suffered from numerous criticisms (e.g. Shiller, 1978 and Fellner, 1980).

From the previous literature two types of test can be found for the rational expectations hypothesis. The first type of rationality test compares the survey data of financial analysts's expectations with actual observations for certain economic variables and then tests the unbiasedness and efficiency of the analysts's forecasts, e.g., Friedman (1980).

However, in addition to the fact that survey data may be unavailable, there are several problems in using survey data to test rationality (see Attfield, Demery, and Duck, 1985). Thus, this type of rationality test is not widely applied in the economic studies.

The second type of rationality test is suggested by Mishkin (1983), which has been applied extensively in macroeconomic studies. This approach conducts the test by comparing a restricted system with an unrestricted system through likelihood ratio test. This paper is worked based on such approach. The test hypothesis and models are provided as follows.

(1) Test Hypothesis:

H_0: The insurer's expectation of growth rate of losses in pricing decision is rational.

(2) Test Model

The test model for the rational expectations hypothesis usually consists of a system of two equations as follows (see appendix 1 for the derivation of testing model):

$$\left.\begin{aligned} \ln L_t &= \beta_0 + \sum_{j=1}^{4} \beta_j \ln L_{t-j} + \sum_{j=1}^{4} \gamma_j \ln X_{t-j} + u_t && \text{(1A)} \\ \ln (P_t/P_{t-1}) &= \alpha_0 + \alpha_1 \{\ln L^e_t - \ln L_{t-1}\} + \alpha_2 \{\ln(1+r_t) - \ln (1+r_{t-1})\} + \epsilon_t && \text{(1B)} \end{aligned}\right\} (1)$$

Where, P_t = price per policy of time t

L_t = mean losses per policy of time t

X_t = economic variable of time t

$\ln L^e_t$ = forecast of $\ln L_t$

$= \beta_0 + \sum_{j=1}^{4} \beta_j \ln L_{t-j} + \sum_{j=1}^{4} \gamma_j \ln X_{t-j}$ for constrained system

{

$= \beta_0 + \sum_{j=1}^{4} b_j \ln L_{t-j} + \sum_{j=1}^{4} c_j \ln X_{t-j}$ for unrestricted system

r_t = discount factor for time t
ϵ_t, u_t = independent random errors at t.
α_i, β_i, γ_i, b_i, c_i = coefficients.

The equation system (1) developed above is applied for automobile liability insurance. The price equation of testing model for automobile physical damage insurance is somewhat revised to include dummy variables for seasonal effects because of significant seasonal variations in physical damage claim costs. Thus the test model for auto physical damage insurance becomes:

$$\ln Lt = \beta_0 + \sum_{j=1}^{4} \beta_j \ln L_{t-j} + \sum_{j=1}^{4} \gamma_j \ln X_{t-j} + u_t \qquad (2A)$$

$$\ln (P_t/P_{t-1}) = \alpha_0 + \alpha_1 \{\ln L^e_t - \ln L_{t-1}\} + \alpha_2 \{\ln(1+r_t) - \ln (1+r_{t-1})\} + \delta_1 D1 + \delta_2 D2 + \delta_3 D3 + \epsilon_t \qquad (2B)$$

}(2)

Where, P_t = price per policy of time t
L_t = mean losses per policy of time t
X_t = economic variable of time t
$\ln L^e_t$ = forecast of $\ln L_t$

$$= \beta_0 + \sum_{j=1}^{4} \beta_j \ln L_{t-j} + \sum_{j=1}^{4} \gamma_j \ln X_{t-j} \text{ for constrained system}$$

{

$$= \beta_0 + \sum_{j=1}^{4} b_j \ln L_{t-j} + \sum_{j=1}^{4} c_j \ln X_{t-j} \text{ for unrestricted system}$$

r_t = discount factor for time t
Di = 1 if the observation is for the season i, otherwise Di = 0.
ϵ_t, u_t = independent random errors at t.
α_i, β_i, γ_i, b_i, c_i = coefficients.

III. Test Methodology

The first step for the rationality test is to specify a rational forecasting model through empirical experiments which applies either univariate time series models or multivariate econometric models (see Mishkin, 1983). The Granger (1969) "causality" concept is a natural way to approach the specification of the multivariate models. The

criterion for specifying the multivariate forecasting equation is as follows. The variable lnLt is regressed on its own four lagged values as well as on four lagged values of a wide-ranging set of macroeconomic variables X.[2] The four lagged values of each of these variables are retained in the equation only if they are jointly significantly different from zero at the 5 percent level, tested by an F statistic.

After the explanatory variables X for the forecasting equation are decided, the method for estimating the rational expectations model involves joint, nonlinear estimations of the equations (1A) and (1B) system with the imposed cross-equation constraints $B_j=b_j$, $\gamma_j=c_j$, j=0,1, ... Then the sum of squared residuals (SSR) from the joint estimated system is taken as SSRc, i.e., the sum of squared residuals of the constrained system.

The next step is to obtain the sum of squared residuals of the unconstrained system, SSR_u. The estimation of the unconstrained system is conducted by estimating the equation (IA) and (IB) respectively without any restrictions on the parameters. The forecasting equation (1A) is estimated through the OLS method, while the decision equation (1B) can be estimated by the OLS estimation or nonlinear GLS[3] with the same explanatory variables as those used in the restricted system (Mishkin, 1983 and Attfield, et al, 1985).

Finally, the test of rationality is performed through the likelihood ratio test. The testing concept is saying that the sum of squared residuals of the two systems would be very close if expectations are rational. The likelihood ratio statistic LR is calculated as follows (see Goldfeld and Quantd, 1972).

$$LR = N \{ \ln (\det \hat{\Sigma}_c) - \ln (\det \hat{\Sigma}_u) \} \qquad (3)$$

where, N = number of observations

det = determinant of a matrix

$\hat{\Sigma}_c$ = the resulting estimated variance-covariance matrix Σ of the residuals of constrained system

$\hat{\Sigma}_u$ = the resulting estimated variance-covariance matrix Σ of the residuals of unconstrained system

Empirically, the values of (det $\hat{\Sigma}$)can be obtained through the sum of squared residuals of each system. That is, equation (3) can be written as equation (4) (Mishkin,

[2] Using four lags is to ensure white noise residuals in the quarterly data (Mishkin, 1983).

[3] Because the parameters in the decision equation appear in nonlinear format, Mishkin (1983) applies nonlinear GLS to estimate the decision equation. However, he also agrees that the OLS can be applied to estimate the decision equation when there are many parameters cannot be identified through the nonlinear estimation.

1983, p19):

$$LR = 2N \{\ln (SSR_c / SSR_u)\} \qquad (4)$$

The LR is distributed asymptotically as $\chi^2(q)$, q is the number of restrictions (Kmenta, 1986), which can be determined by subtracting the number of the identified parameters of the restricted system from the number of identified parameters of the unrestricted system.

The hypothesis of rational expectations is tested by compared LR with the χ^2 value with q degree of freedom. If LR is greater than $\chi^2(q)$ significantly at 5 percent level, the hypothesis would be rejected; otherwise, we cannot reject the hypothesis.

IV. Data Description

Due to data constraints, the empirical tests are conducted only for private passenger automobile insurance in the U.S.. Quarterly data are applied in the tests. The test period extends from the first quarter of 1975 (1975.1) to the third quarter of 1988 (1988.3).

The rationality test is applied to two types of insurance products, private automobile liability and private automobile physical damage insurance. The automobile liability insurance is a long-tail line of insurance and physical damage insurance is a short-tail line. The application of empirical tests to the two insurance products provides some comparisons of underwriting results between long-tail lines and short-tail lines.

Data for premiums (price) are from Best's Quarterly By Line data file which provides data on total country-wide premium written for the industry for private passenger auto liability insurance and auto physical damage insurance in each quarter for the period from 1975.1 to 1988.3. Data in this file are obtained by A.M.Best Co through questionnaire and account for about 80 percent of the premium volume in the U.S. property-liability insurance industry.

Data on losses (L_t) are obtained from the Fast Track data base of National Association of Independent Insurers (NAII). This data file contains private passenger auto insurance quarterly data of industry-wide claims frequency and average paid claim cost (severity) for each type of coverage from the first quarter of 1975 to the third quarter of 1988. Data are reported to Fast Track by all companies that are members of the Insurance Services Office (ISO) and NAII. Together these companies account for about 90 percent of auto insurance market share in the U.S.

The average three-month Treasury Bill rate for each quarter is substituted for data on the discount factor forecast (r_t). Treasury Bill rate data are obtained from the Board of Governors of the Federal Reserve Board.

The data for macroeconomic variables for the same period (1975.1-1988.3) are obtained from the Consumer Price Index Detailed Report, a publication of US Department of Commerce. The macroeconomic variables chosen for the empirical analysis include general consumer price index (CPI), hourly wage rate, medical expense CPI, new car CPI, used car CPI, and automobile insurance CPI, because these variables are considered to be most relevant to automobile insurance coverage.

V. Test Results

(1) Private Automobile Liability Insurance

Since the rationality test is applied to two insurance products, the results are provided separately. The empirical results based on private automobile liability insurance are discussed first.

The specification of the forecasting equation is based on the results of empirical experiments on univariate and multivariate time series models. Finally the forecasting equation is specified as a multivariate model by way of Granger procedures outlines above.[4]

The CPI of medical expense is selected as the economic variable X because the F statistic for the joint test of its four lagged values is the most significant one among the chosen economic variables (see table 1). Consequantly, the explanatory variables finally selected in the forecast equation to test the rational expectation hypothesis include four lagged values of the logarithm of L_t as well as the four lagged values of the logarithm of the CPI of medical expenses.

The empirical results of parameter estimates of the price equation, under the nonlinear joint estimation of equations (1A) and (1B) with cross-equation restrictions

[4] According to the Granger "causality" concept, every variable which can increase the prediction power should be included in the forecasting equation. In empirical studies of rational expectations, a variable is chosen for the forecasting equation once the F value for the four lagged values of that variable is significantly different from zero at 5 percent level (Mishkin, 1983). It is possible to include several economic variables in the forecasting equation if this selection formula is completely applied.
However, some modification is needed because of the small sample size in this paper. Consequently this paper includes at most one economic variable in the forecasting equation. The variable selected is the one whose F value is most significantly different from zero.

Table 1

F Statistics for Significant Explanatory Power in Forecasting Equation of Four Lags of Each Variable

$$\text{Model: } \ln L_t = \beta_0 + \sum_{j=1}^{4} \beta_j \ln L_{t-j} + \sum_{j=1}^{4} \gamma_j \ln X_{t-j} + u_t$$

Variable (X)	Auto Liability	Auto Physical Damage
General CPI	2.499	1.059
Wage Rate	1.189	0.595
Medical CPI	3.932**	1.263
New Car CPI	2.819*	2.951*
Used Car CPI	3.168*	0.404
Auto Ins. CPI	1.001	1.569

Note: the numbers in table 1 are the F statistics for testing $\gamma_1 = \gamma_2 = \gamma_3 = \gamma_4 = 0$ of the forecasting equation.
* = significant at 5 percent level F (4,41, 0.05) = 2.60
** = significant at 1 percnet level F (4,41, 0.01) = 3.82

$\beta_1=b_1$ and $\gamma_1=c_1$, are shown in table 2.[5] The results indicate that the price changes are affected by the forecast of loss inflation rates but not significantly related to the changes in interest rates.

The sum of squared residuals for the restricted system is equal to 0.1383, and the sum of squared residuals for the unrestricted system is 0.1264. Consequently, the likelihood ratio LR between the restricted system and unrestricted system is equal to 8.99. It is less than the critical value $\chi^2(7)$ at 5 percent level which is 14.067. Therefore, the hypothesis that the insurer's forecasts of the loss inflation rates are based on rational expectations cannot be rejected for the private automobile liability insurance.

The parameter estimates of forecasting equation under the jointly estimations of

[5] Because in practice there probably exists information lag for liability insurance, the losses experience of t-1 is not available for price of t. The data available at the end of t-1 for price of t are in fact the losses experienced in t-2. Thus the empirical analysis is conducted on one lagged data of the explanatory variables for the price equation (1B).

Table2

Rationality Test for Private Automobile Liablity Insurance

Model: $\ln(P_t/P_{t-1}) = \alpha_0 + \alpha_1 \{\ln L^e_t - \ln L_{t-1}\} + \alpha_2 \{\ln(1+r_t) - \ln(1+r_{t-1})\} + \epsilon_t$

parameter	estimate	t' value[6]
α_0	-0.0009	(-0.105)
α_1	1.3880	(6.309)*
α_2	-0.4665	(-0.892)
SSR_c	0.1383	
SSR_u	0.1264	
Likelihood Ratio	8.9973	

Note: * = significant at 0.05 level

$t(47, 0.05) = 2.01$, $\chi^2(7, 0.05) = 14.067$

N = 50

$\ln L^e_t = \beta_0 + \sum_{j=1}^{4} \beta_j \ln L_{t-j} + \sum_{j=1}^{4} \gamma_j \ln X_{t-j}$

X = medical expense CPI

the restricted system and the OLS estimates of the forecasting equation of unrestricted system (i.e., rational forecast) are provided in table 3 for the purpose of comparison. The results under two systems are similar, which also supports the rationality hypothesis.

(2) Private Automobile Physical Damage Insurance

The same methodology of rationality test is also applied to the private automobile physical damage insurance which is one of the short-tail insurance products in the property-liability insurance industry.

Again, the first step is to specify the forecasting equation based on empirical

[6] Since we have not been able to determine the small sample properties of the parameter estimators of nonlinear regression models, we clearly cannot hope to obtain precise confidence interval. However, we may, for simplicity, assume that in the nonlinear regression model the error terms are normally distributed. In analogy to t-ratio in the case of linear regression, we calculate the ratio of coefficient estimate to its asymptotical standard error and use t distribution as approximation to the actual distribution (see Judge, et al, 1982, p657). Consequently, this paper simply denotes the ratio of coefficient estimate to its standard error by t' value.

Table 3

Parameter Estimates of the Forecasting Equation for Private Automobile Liability Insurance

$$\text{Model:} \ln L_t = \beta_0 + \sum_{j=1}^{4} \beta_j \ln L_{t-j} + \sum_{j=1}^{4} \gamma_j \ln X_{t-j} + u_t$$

variables	restricted system	OLS estimates
intercept	–0.0592 (–0.3572)	0.0628 (0.2910)
$\ln L_{t-1}$	0.3975* (3.8480)	0.3689* (2.8865)
$\ln L_{t-2}$	–0.0033 (–0.0375)	–0.1448 (–1.2408)
$\ln L_{t-3}$	0.0215 (0.1987)	0.1508 (1.0436)
$\ln L_{t-4}$	0.5277* (4.9272)	0.4028* (2.9837)
$\ln X_{t-1}$	0.3974 (0.4579)	1.4939 (1.2667)
$\ln X_{t-2}$	–2.2258 (–1.4185)	–3.8371 (–1.8073)
$\ln X_{t-3}$	4.7044* (3.1735)	5.0850* (2.6328)
$\ln X_{t-4}$	–2.7960* (–3.3928)	–2.5097* (–2.3116)
R^2	0.9922	0.9929

Note: X = medical expense CPI, N = 50

* = significant at 0.05 level

t values in parentheses, t (41, 0.05) = 2.02

experiments on univariate and multivariate time series models. The final version of the forecast equation is specified using the multivariate model by way of Granger procedures.

The explanatory variables finally selected in the forecast equation needed to test the rational expectation hypothesis include four lagged values of the logarithm of L_t as well as four lagged values of the logarithm of new car CPI because the F statistic of the new car CPI is the most significant among the economic variables (see table 1).

Table 4 shows the empirical results of parameter estimates of the price equation under the nonlinear joint estimation of equations (2A) and (2B) with cross-equation restrictions $\beta_i=b_i$ and $\gamma_i=c_i$. The results suggest that the price changes of auto physical

Table 4

Rationality Test for Private Automobile Physical Damage Insurance

Model: $\ln(P_t/P_{t-1}) = \alpha_0 + \alpha_1 \{\ln L^e_t - \ln L_{t-1}\} + \alpha_2 \{\ln(1+r_t) - \ln(1+r_{t-1})\} + \delta_1 D1 + \delta_2 D2 + \delta_3 D3 + \epsilon_t$

parameter	estimate	t' value
α_0	–0.0571	(–4.8390)*
α_1	0.4035	(2.8576)*
α_2	–0.1640	(–0.3789)
δ_1	0.1546	(9.9103)*
δ_2	0.0732	(3.5023)*
δ_3	0.0781	(5.3129)*
SSR_c	0.0947	
SSR_u	0.0908	
Likelihood Ratio	4.2055	

Note: * = significant at 5 percent level, N = 50

$t(44, 0.05) = 2.01$, $\chi^2(7, 0.05) = 14.067$

N = 50

$\ln l^e_t = \beta_0 + \sum_{i=1}^{4} \beta_i \ln L_{t-i} + \sum_{i=1}^{4} \gamma_i \ln X_{t-i}$

X = new car CPI

damage insurance are related to the forecast of loss inflation rates but not significantly related to the changes in interest rates.

The sum of square residuals for each system are also reported in table 4. The SSR of the restricted system is equal to 0.0947, and the SSR of the unrestricted system is 0.0908. Thus, the likelihood ratio LR between the restricted system and unrestricted system is equal to 4.205. It is less than the critical value $\chi^2(7)$ at 5 percent level, 14.067. Therefore, the hypothesis that the insurer's forecasts of the loss inflation rates are based on rational expectations cannot be rejected for private automobile physical damage insurance.

Furthermore, the parameter estimates of forecasting equation under the jointly estimations of the restricted system and the OLS estimates of the forecasting equation of unrestricted system (i.e., rational forecast) are provided for the purpose of comparison. Table 5 shows that the parameter estimates and their significance are similar under two systems, which support the rationality hypothesis.

The empirical results in this section suggest that the rational expectations hypothesis for insurance rate-making decision cannot be rejected for either the automobile liability insurance, one of the long-tail line insurance products, or the automobile physical damage insurance, one of the short-tail line insurance.

VI. Conclusion and Suggestion

This paper has aimed to provide an empirical analysis of rate-making decisions for the property-liability insurance industry to explain the causes of underwriting cycles. This study tests the hypothesis that the insurance rate-making decision is based on rational expectations. The test results suggest that the rational expectations hypothesis cannot be rejected for either private automobile liability insurance or physical damage insurance.

The testing results of this paper offer some supports for previous studies which argued that underwriting cycles might be caused by uncontrollable (institutional) factors instead of insurer's irrational decision process. It implies that the approach to alter underwriting cycles should be through changing institutional factors.

However, the rationality test methodology applied in this paper is only one of the alternatives for testing the rational expectations hypothesis. There are several limitations on this methodology. For example, this type of test applies only to the validity of the cross equation constraints under the maintained model. It cannot distinguish whether the structures of those equations are true models. (Abel and Mishkin, 1983). Several

Table5

Parameter Estimates of the Forecasting Equation
for Private Automobile Physical Damage Insurance

$$\text{Model: } \ln L_t = \ln L_t = \beta_0 + \sum_{j=1}^{4} \beta_j \ln L_{t-j} + \sum_{j=1}^{4} \gamma_j \ln X_{t-j} + u_t$$

variables	restricted system	OLS estimates
intercept	–0.5047 (–1.1878)	–0.7413 (–1.5408)
$\ln L_{t-1}$	0.3256* (2.5862)	0.3384* (2.4647)
$\ln L_{t-2}$	–0.0638 (–0.5768)	–0.1075 (–0.9149)
$\ln L_{t-3}$	0.1791 (1.4888)	0.1328 (1.0367)
$\ln L_{t-4}$	0.4191* (3.3690)	0.4197* (3.0838)
$\ln X_{t-1}$	2.4234* (3.3220)	2.1748* (2.6626)
$\ln X_{t-2}$	–3.5720* (–3.2973)	–3.2802* (–2.7029)
$\ln X_{t-3}$	2.2561* (2.2249)	2.5758* (2.2581)
$\ln X_{t-4}$	–0.8816 (–1.3103)	–1.1307* (–1.4846)
R^2	0.9788	0.9791

Note: * = significant at 0.05 level, X = new car CPI
t values in parentheses, t (41, 0.05) = 2.02

different approaches are also available for rationality tests. Future studies may adopt different approaches to reexamine market rationality to provide a comparison of the findings.

Appendix 1

Price equation is developed based on the basic concept of fair rate principle that pure premium is equal to present value of losses. However, the loss settlement delays in insurance practice make the equation somewhat complicated which is discussed by the following two cases.

Case 1: No loss settlement delays (short-tail lines like auto physical damage insurance are similar to this type)

Price equation is:

$$P_{t+1} = L_t\,(1+g_{t+1}) / (1+r_{t+1})$$

where, $(1+g_{t+1}) = L^e_{t+1}/L_t$

To match with the forecast equation, we take log transformation:

$$\ln P_{t+1} = \ln L_t + \ln (1+g_{t+1}) - \ln (1+r_{t+1}) \qquad \text{(A1)}$$

Then,

$$\begin{aligned}
\ln P_{t+1}/P_t &= \ln \{[L_t/L_{t+1}] / (1+g_t)\}\ \{1+g_{t+1}\}\ \{(1+r_t) / (1+r_{t+1})\} \\
&= \ln \{L_t/L^e_t\}\{L^e_{t+1}/ L_t\}\ \{(1+r_t) / (1+r_{t+1})\} \\
&= \ln \{L_t/L^e_t\}\{L^e_{t+1}/ L_t\}\ \{(1+r_t) / (1+r_{t+1})\} \\
&= \{\ln L_t - \ln L^e_t\} + \{\ln L^e_{t+1}\ \ln L_t\} - \{\ln(1+r_{t+1}) - \ln (1+r_t)\} \\
&= u_t + \{\ln L^e_{t+1} - \ln L\} + \{\ln(1+r_{t+1}) - \ln (1+r_t)\} \qquad \text{(A2)}
\end{aligned}$$

Therefore, the testing equation is:

$$\ln P_{t+1}/P_t = \alpha_0 + \alpha_1\ \{\ln L^e_{t+1} - \ln L_t\} + \alpha_2\ \{\ln(1+r_{t+1}) - \ln (1+r_t)\} + \epsilon_{t+1}$$

Case 2: With loss settlement delays

Price equation in such case is like:

$$P_{t+1} = L_t\{ \sum_{i=1}^{n} \theta_i(1+g_{t+1})^i / -(1+r_{t+1})^i\}$$

Where, θ_i= proportion of loss settlement in development period i

Due to the technical restrictions of the rationality test, direct testing on the above equation is impossible. Some approximations for the power series are needed to derive the testing equation.

By way of totally differentiating the price equation with respect to L, (1+g), and (1+r), and then dividing the differentiation by the price equation, we get that the percentage change in price is approximately equal to the sum of percentage changes in L and (1+g) minus percentage change in (1+r). This is based on the method of duration for bond returns in financial theory (see Sharpe, 1985).

That is,

$$dP/P \approx \{dL/L\} + D\{d(1+g) / (1+g)\} - D\{d(1+r) / (1+r)\} \qquad \text{(A3)}$$

where, d = notation of differentiation

$$D = \{ \sum_{i=1}^{n} i \ \theta_i(1+g)^i / (1+r)^i\} / \{ \sum \theta_i(1+g)^i / (1+r)^i\} \}$$

$$= \text{duation}$$

The equation (A3) is equivalent to the total differentiation of a logarithm function like:

$$\ln P = \ln L + D\ln(1+g) - D\ln(1+r) \qquad \text{(A4)}$$

Therefore, the equation (A4) is taken as the approximate pricing equation for long-tail line insurance in order to develop the testing equation. Thus,

$$\ln P_{t+1} = \ln L_t + D\ln(1+g_{t+1}) - D\ln(1+r_{t+1}) \qquad \text{(A5)}$$

$$\begin{aligned}
\ln P_{t+1} - \ln P_t &= \ln L_t + D\ln(1+g_{t+1}) - D\ln(1+r_{t+1}) - \ln L_{t-1} - D\ln(1+g_t) + D\ln(1+r_t) \\
&= \{\ln L_t - \ln L_{t-1} - D\ln(1+g_t)\} + D\{\ln(1+g_{t+1})\} \\
&\quad - D\{\ln(1+r_{t+1}) - (1+r_t)\} \\
&= \{\ln L_t - \ln L_{t-1} - D(\ln L^e_t - \ln L_{t-1})\} + D\{\ln(1+L^e_{t+1}) - \ln(1+L_t)\} \\
&\quad - D\{\ln(1+r_{t+1}) - \ln(1+r_t)\} \qquad \text{(A6)}
\end{aligned}$$

Since at the beginning of t+1 the L_t, L_{t-1} and L^e_t are all known information, the first item in equation (A6) is just to reflect the random errors (u_t) of previous forecast. Thus the euqation (A6) can be rewritten as equation (A7):

$$\ln P_{t+1} / P_t = C(u_t) + D \{ \ln (1+L^e_{t+1}) - \ln (1+L_t) \} - D \{ \ln (1+r_{t+1}) - \ln (1+r_t) \} \quad (A7)$$

where, C and D are approximately constant

The interpretation of equation (A7) is similar to equation (A2) and thus the testing equation is the same as before.

REFERENCES

1. *A. Abel and F. Mishk in (1983)*, "An Integrated View of Tests of rationality, Market Efficiency and the Short-Run Neutrality of Monetary Policy," Journal of Monetary Economics, pp 3-24.
2. *C. Attfield, D. Demery and N. Duck (1985)*, "Rational Expectations in Macroeconomics," Basil Blackewll, New York.
3. *C. Attfield, and N. Duck (1982)*, "Tests of rational Expectations Model of the Term Structure of UK Interest Rates," Economics Letters, pp 115-121.
4. *R. Barro (1977)*, "Unanticipated Money Growth and Unemployment in the United States," American Economic review, pp 101-115.
5. *D. Cummins and F. Outreville (1987)*, "An International Analysis of Underwriting Cycles in Property-Liability Insurance," Journal of Risk and Insurance, pp 246-262.
6. *N. Doherty and H. Kang (1988)*, "price Instability for a Financial Intermediary: Interest rates and Insurance Price cycles," Journal of Banking and finance.
7. *W. Fellner (1980)*, "The valid Core of rationality Hypotheses in the Theory of expectations," Journal of Money, Credit and Banking, pp 763-87.
8. *B. Friedman (1980)*, "Survey Evidence on the 'Rationality' of Interest Rate Expectations," Journal of Monetary Economics, pp 753-766.
9. *D. Givoly (1985)*, "The Formation of Earnings Expectations," The Accounting Review.
10. *S.Goldfeld and R.Quandt (1972)*, "Non-Linear Methods in Economerics," Amsterdam, North-Holland.
11. *C. Granger (1969)*, "Investigating Causal Relations by Econometric Models and Cross-Spectral Methods," econometrica, 37, pp 424-538.
12. *J. Kmenta (1986)*, "Elements of Econometrics", 2nd ed. Macmillan Publishing Co, New York.
13. *F. Mishkin (1982)*, "Does Anticipated Aggregate Demand Policy Matter? Further Econometric Results," American economic Review 72.
14. *F. Mishkin (1983)*, "A Rational Expectations Approach to Macroeconometrics", NBER monograph, U of Chicago Press, Chicago.
15. *J. Muth (1961)*, "Rational Expectations and the Theory of price Movements," Econometrica, 1961, pp 315-35.
16. *W. Sharpe (1985)*, "Investments," Prentice-Hall, Inc. New Jersey.
17. *R. Shiller (1978)*, "Rational Expectations and the Dynamics Structure of Macroeconomic Models," Journal of Monetary Economics, pp 1-44.
18. *B. Stewart (1984)*, "Profit Cycles in Property-Liability Insurance." In J. Long, ed. Issue in Insurance, Malvern, PA, American Institute for Property-Liability Underwriters.
19. *C. Wilson (1981)*, "The Underwriting Cycle and Investment Income." CPCU Journal, pp 225-232.